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Laboratories and Demonstrations

A Variation of the Speed of Sound Experiment

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An experiment which is familiar to most physical chemistry students is the speed of sound based on the Kundt's tube.[l] The experiment also allows the student to determine the heat-capacity ratio for various gases. The process of changing gases may be more easily accomplished if the microphone in the tube can be fixed in its position relative to the speaker. In this article, this modification to the experiment is discussed in terms of resonance generated in closed pipes.

The determination of the speed of sound in gases is a standard physical chemistry experiment [l, 2]. From the speed of sound, the heat capacity ratio of a gas can be calculated. The heat capacity ratio γ is C_p/C_v in which C_p, and C_v are the heat capacities measured at constant pressure and volume,

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respectively. Although the ratio can also be obtained by the adiabatic expansion method [3], the ease with which the speed-of-sound experiment can be performed makes this the method of choice for most instructors. The speed of sound in a gaseous medium is a function of this heat-capacity ratio as well as the average speed of the molecules moving in the medium. The average speed is in turn a function of the molar mass and the temperature, *M* ant *T*, respectively, and hence the speed of sound in gases is given by [4]:

$$s = \sqrt{\frac{\gamma RT}{M}}$$

In order to determine the velocity of sound in the typical experiment, a modified version of Kundt's tube is often used [l, 2]. Here, a speaker is placed at the end of a tube and a moveable microphone is positioned within the tube. A portion of the signal from an audio-frequency generator is fed to the speaker while the other part is fed to one of the axes of an oscilloscope. The scope is configured to receive the signals in the *xy* mode. The speaker generates standing waves within the tube that are picked up by the microphone. The microphone's output serves as the input to the other axis of the oscilloscope, resulting in a Lissajous pattern. As the microphone is moved along the length of the tube, the Lissajous pattern changes, reflecting the relative phase between the sine wave that drives the speaker and the amplitude of the pressure wave in the tube that is being detected by the microphone. If the microphone is moved from one in-phase pattern to the next in-phase pattern, the distance that the microphone was translated represents one full wavelength of the audio frequency. By noting the frequency of the audio signal v and its wavelength λ, the speed of sound s can be calculated using the simple equation:

$$s = \lambda \times v \quad (1)$$

Note that the translation can be done from 0° in-phase to 180° out-of-phase, or from 0° in phase to 90° out-of-phase. In these cases the length needs to be multiplied by factors of 2 and 4, respectively, to account for the half and quarter wavelength being measured. This method in which the wavelength is calculated by translating the microphone is how the experiment is typically done.

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A variation to this experiment is to fix the distance between the microphone and speaker, and to sweep the audio frequency from one in-phase to the next in-phase Lissajous pattern. Now note that for any arbitrary audio frequency ν_1 for which an in-phase Lissajous pattern is observed, its wavelength is given by:

$$\lambda_1 = \frac{d}{n_1} \tag{2}$$

in which d is the distance between the speaker and the microphone and n_1 represents the first harmonic being generated. Keeping the distance fixed, the frequency is increased until the 180° out-of-phase Lissajous pattern is observed, whereupon at this frequency, designated as ν_2, the wavelength is now given by:

$$\lambda_2 = \frac{d}{n_2} \tag{3}$$

The explanation for the alternation in the phase angle will be explain later in the text. When equation 1 is substituted into equations 2 and 3, we obtain:

$$s = \left(\frac{d}{n_1}\right)\nu_1 \qquad \text{and} \qquad s = \left(\frac{d}{n_2}\right)\nu_2$$

If we now multiply the first equation by n_1 and the second by n_2, and if we were to subtract these two relationships:

$$s = d\,\frac{(\nu_2 - \nu_1)}{(n_2 - n_1)} \tag{4}$$

The propagation of a wave at any boundary causes the wave to be reflected. Constructive interference occurs when the amplitude node occurs at this boundary. An analysis of the wave pattern in the tube indicates that this tube is an air column, similar to an organ pipe, and more specifically, a closed tube. In such a tube, only the odd harmonics are present [5, 6]. In Figure 1, the amplitudes for several odd harmonics are plotted as a function of distance from the microphone to the speaker. Some deviation from ideal behavior is observed at the opening [6].

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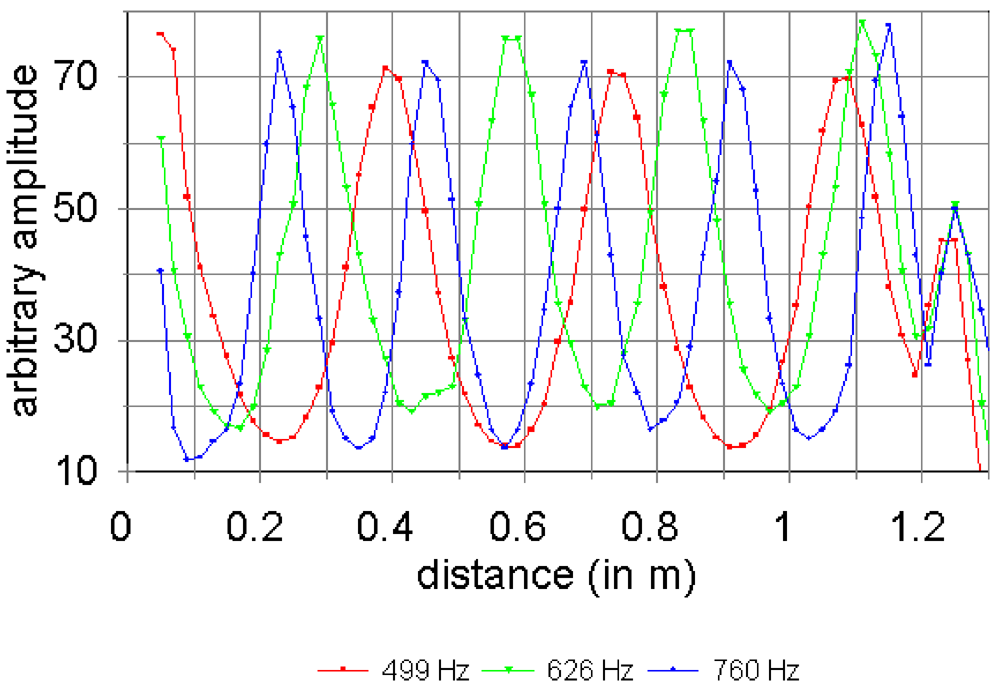

FIGURE 1. AMPLITUDES OF THE STANDING WAVES AS A FUNCTION OF THE DISTANCE FROM THE SPEAKER TO THE MICROPHONE FOR THE 7TH, 9TH, AND 11TH HARMONIC OVERTONES FOR THE CLOSED TUBE. THE LENGTH OF THE TUBE IS 1.25 m.

The first harmonic, or the fundamental of this closed tube is given by:

$$\nu_1 = \frac{s}{(4L)} \tag{5}$$

in which L is the length of the tube [5, 6]. The factor of 4 arises because, in the closed tube, constructive interference first occurs for the quarter wave. As noted above, for a closed tube constructive interference favors only the *odd* harmonics; hence, the next resonant frequency is the third harmonic, given by:

$$\nu_3 = \frac{3s}{4L}$$

If the frequency is swept carefully as to not miss any intervals between adjacent Lissajous patterns, the next in-phase signal must come when the denominator of equation 4 is even:

$$n_2 - n_1 = (n_1 + 2) - n_1 = 2$$

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Consequently, for adjacent frequencies, which give rise to the next in-phase Lissajous pattern, the speed of sound is given by:

$$s = \left(\frac{d}{2}\right)(\nu_2 - \nu_1) = 2L(\nu_2 - \nu_1) \qquad (6)$$

in which we used the fact that the fundamental frequency is a quarter wave in a closed tube, and if the microphone is placed at the end of the tube:

$$d = 4L$$

For a closed tube, the signal at the first minimum in the standing wave of the pressure amplitude is phase-shifted by 180° relative to the signal at the speaker. The speaker, representing the closed end of the tube is at the antinode and is in-phase until the first node. This means that for the fundamental the phase angle measured at the open end of the tube is 180° phase-shifted. For each of the successive overtones the phase angle at the open end of the tube alternates between 0° and 180° because the number of minima increases by one. Using this method, the resonant frequencies for air in a closed tube are shown in Table 1.

An alternate way of doing this experiment is to fix the microphone at one-half the length of the tube. In this case the Lissajous pattern will periodically alternate in- and out-of-phase by 0°, 90° , 180°, then back to 0°. This can be observed from Figure 1. In practice, the position of the microphone that gives rise to the Lissajous pattern indicating out-of-phase by 90°, a circle, is difficult to determine precisely. Therefore, if only the frequencies at 0° and 180° phases are used, as can be seen from Figure 1, at $L/2$, every other odd harmonic is observed. Referring to equation 4, the 0° and 180° patterns occur when $n_2 = 4\ n_1$; and hence, if L is the length of the entire tube, equation 6 may be used by leaving out the factor of 2.

It should be noted that the position of the microphone is very important; the most convenient positions are at the open side of the tube, or half the tube length. Although other positions are possible, such as higher even fractions of the tube length, for example, 1/4 and 1/8, the accurate positioning of the microphone will become increasingly critical. The point is that the microphone may not be at any arbitrary distance inside the tube. This can be more easily explained using Figure 2, which shows the pressure amplitude within the tube for frequencies slightly off resonance.

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TABLE 1. Calculated and experimental frequencies for the odd harmonics in the closed tube, measured using the fixed distance method. The calculated fundamental frequency was determined from equation 4 using 344 m s^{-1} as the speed of sound in air and L = 1.25 m. $\Delta\nu$ is the difference in the experimental frequencies between the row that it is in and the row below. The average $\Delta\nu$ is reported at the 95% confidence interval. An asterisk indicates that the frequency response of the microphone did not allow this measurement. The average experimental speed of sound using equation 6 and the average $\Delta\nu$ is 344 ± 7.7 m s^{-1}.

Harmonic number	Calculated	Experimental	$\Delta\nu$
1	69	*	
3	207	210	140
5	345	350	149
7	483	499	127
9	621	626	134
11	759	760	136
13	897	896	144
15	1035	1040	138
17	1173	1178	138
19	1311	1316	137
21	1449	1453	134
23	1589	1587	143
25	1725	1730	132
27	1863	1862	134
29	2001	1996	142
31	2139	2138	
		Average $\Delta\nu$	**138 ± 3.0**

The transition between consecutive overtones, as a result of constructive interference, as a function of frequency is not a smooth one, as evidenced from this figure.

One may also close the open side of this tube and place the microphone either on the end or inside the tube [2]. This case is not normally treated in acoustics; however, a plot of the standing waves generated by such a tube for the fundamental and second harmonic is shown in Figure 3. What is apparent is that for the tube, which is closed on

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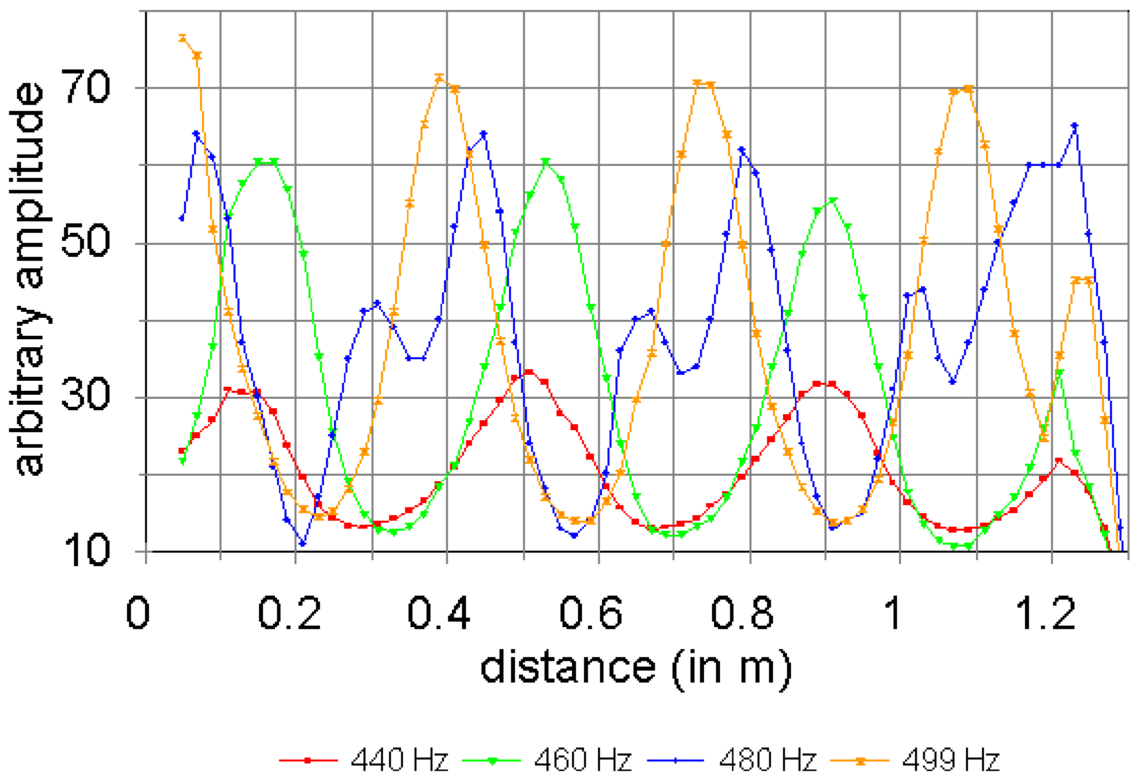

FIGURE 2. AMPLITUDES OF THE STANDING WAVES AS A FUNCTION OF THE DISTANCE FROM THE SPEAKER TO THE MICROPHONE FOR SEVERAL FREQUENCIES NEAR THE 7TH HARMONIC FOR THE CLOSED TUBE.

both sides, the fundamental wavelength $\lambda = 2L$, which is similar to the open tube, that is the tube which is opened on both sides [5, 6]. Note that if the microphone is placed at the end of the tube then the signals are always in-phase relative to the speaker. On the other hand, as can be inferred from the figure, if the microphone is placed inside the tube at $L/2$ then the phase angle will alternate between 180 and 0 degrees.

As a comparison of the methods is as follows. With 50 replicate measurements and at the 95% confidence level, the moving microphone method gave a result of 342 ± 6.0 m s^{-1} as the speed of sound in air. With the microphone positioned at the end of the tube and the tube closed on both ends, the velocity of sound was found to be 342 ± 4.0 m s^{-1} using 40 measurements. From Table 1, the closed tube with the microphone positioned at the end of the tube yielded 344 ± 7.7 m s^{-1}. The speed of sound in air is 344 m s^{-1} at 22 °C, the temperature of the room in which the experiment was performed [5]; therefore, all of these methods yield comparably accurate and precise results.

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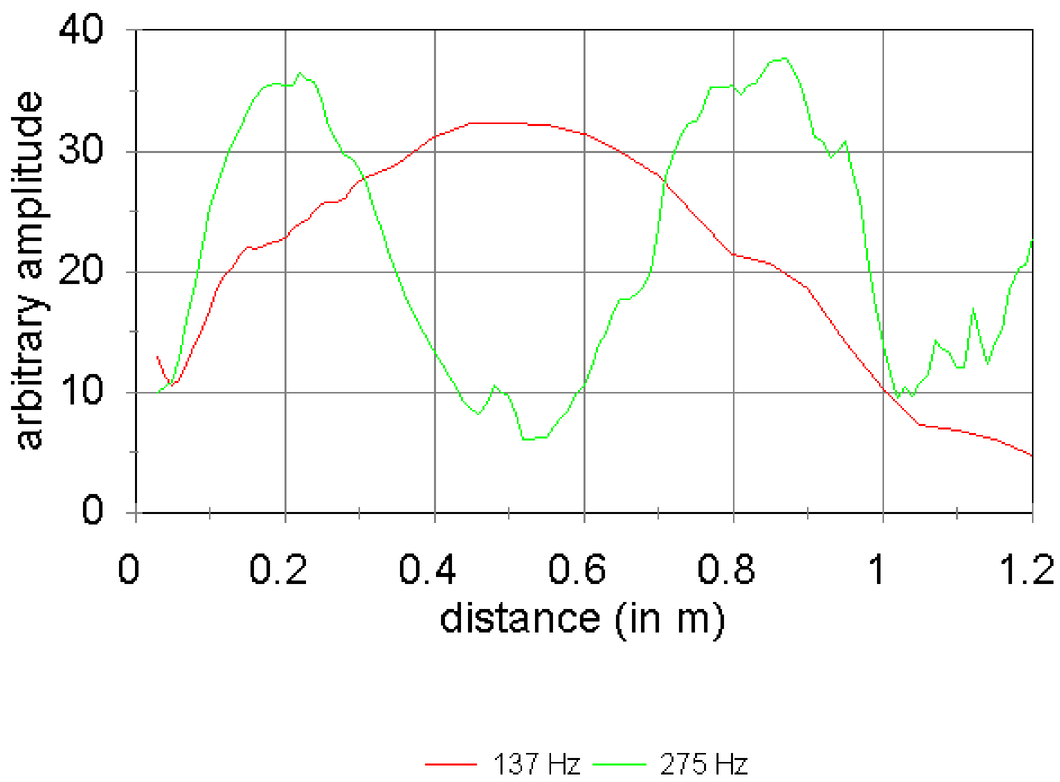

FIGURE 3. THE AMPLITUDE OF THE STANDING WAVE AS A FUNCTION OF THE DISTANCE FROM THE SPEAKER TO THE MICROPHONE FOR THE FUNDAMENTAL AND SECOND HARMONIC OF A TUBE IN WHICH A SPEAKER HAS BEEN PLACED ON ONE SIDE AND CLOSED ON THE OTHER. THE LENGTH OF THE TUBE IS 1.25 m.

One notable advantage of the fixed microphone tube is that the method does facilitate the use of different gases in the Kundt's tube apparatus. The disadvantage of the moving microphone method is that once a gas has been introduced into the tube, it's difficult to move the microphone without introducing air into the system. With the fixed microphone method, if one carefully measures the outside length of the tube then places the microphone either at the end or, alternatively, at half of this length, then the microphone can be rigidly and securely positioned. For the closed tube the gas can be introduced into a vertically positioned tube, in which the open end is pointed either up or down, depending upon the density of the gas relative to air. The tube that is closed on both ends can be purged, filled with the gas, and sealed. A fixed-configuration idea is also used in the spherical resonator method described by Colgate et al. [7], but if the Kundt's tube is already in use, no further modification is required to apply the method described here.

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